Detection of Targets Embedded in Multipath Clutter with Time Reversal

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***Abstract*—Detection of targets in complex environments is of importance in both radar and sonar applications. Recent work has shown that the use of Time Reversal (TR) techniques improves the performance of systems operating in deterministic channels with a significant multipath return. This paper extends those results to stationary random channels with significant multipath. We develop a TR-based approach and derive the Likelihood Ratio Test (LRT) for this approach. We compare this TR-LRT to an LRT derived through a "water filling" approach. We derive theoretical performance curves for the water filling LRT, and evaluate both the water filling and TR detectors with Monte Carlo simulations. For the scenarios tested, we show that TR achieves an SNR gain of $1-2$dB over the water filling detector.**

***Index Terms*—Time Reversal, Detection, Multipath, Radar, Water Filling.**

I. Introduction

Time Reversal (TR) is an adaptive waveform transmission technique that can be used in a non-homogenous transmission medium to focus energy on a particular point in time and space [1]. TR assumes reciprocity in the transmission medium and provides transmit signals that approximate a channel matched filter, without the need to model complex environments explicitly. Although originally developed for optical and acoustic applications, it has been given recent attention in electromagnetics [2], [3], [4]. While some prior work for TR detection has focused on deterministic channels that allow for removal of clutter through change detection, we extend these results to a stationary random channel model.

We consider the problem of detecting a target embedded in stationary random multipath clutter. We derive a detection scheme, based on TR, to detect the presence of a target in the ideal case, where the second order statistics are all known (or can be learned). For a comparison, we also present an alternative, based on the results of [5], which proved the classic "water filling" strategy to be optimal for targets with a flat PSD; we deem this to be the conventional approach. The water filling strategy attempts to minimize the strength of the clutter response, by allocating power to each frequency in proportion to the inverse of the clutter PSD. We present the problem description, and Likelihood Ratio Test (LRT) in Section II. The water filling and TR approaches are both outlined in Section III. We present Monte Carlo simulations for two representative scenarios in Section IV, where we discuss the effectiveness of TR. Our conclusion is given in Section V.

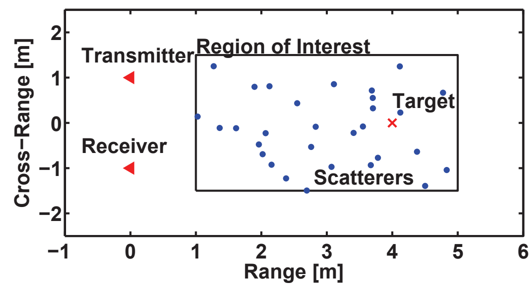

Fig. 1. Graphical Depiction of Problem Scenario.

II. Problem Formulation

We consider the detection of a target embedded in stationary random multipath scattering, using a single transmitter and receiver. See figure 1 for a graphic of one possible scenario. The received signal is broken into three components: $C(\omega)$ is the signal caused by reflections from the clutter, $T(\omega)$ is the response induced by the presence of the target (this includes direct-path, as well as multi-path signals that involve the target,) and $V(\omega)$ is sensor noise. We denote the null hypothesis to be the case where the target is not present ($T(\omega) = 0$). Thus, the received signal $Y(\omega)$ can be stated by the binary hypothesis:

$$\begin{array}{lcl} \mathcal{H}_0 & : & Y(\omega) = C(\omega) + V(\omega) \\ \mathcal{H}_1 & : & Y(\omega) = T(\omega) + C(\omega) + V(\omega) \end{array} . \quad (1)$$

The system is sampled at frequencies $\omega_q, q \in [0, Q-1]$, with the sampling chosen such that each frequency bin is separated by the coherence bandwidth of the channel [6]; this provides independence of the samples across frequency. We assume that $T(\omega_q)$, $C(\omega_q)$, and $V(\omega_q)$ are all complex Gaussian random variables, and that they are independent of each other. This will be discussed in the following section. We define the vector

$$\mathbf{y} = [Y(\omega_0), \ldots, Y(\omega_{Q-1})]^T , \quad (2)$$

and similarly stack the frequencies of the target, clutter, and noise responses to obtain $\mathbf{t}$, $\mathbf{c}$, and $\mathbf{v}$, respectively. Thus, we rewrite the binary hypothesis problem:

$$\begin{array}{lcll} \mathcal{H}_0 & : & \mathbf{y} = \mathbf{c} + \mathbf{v} & \sim \mathcal{CN}(\boldsymbol{\mu}_0, \boldsymbol{\Gamma}_0) \\ \mathcal{H}_1 & : & \mathbf{y} = \mathbf{t} + \mathbf{c} + \mathbf{v} & \sim \mathcal{CN}(\boldsymbol{\mu}_1, \boldsymbol{\Gamma}_1) \end{array} , \quad (3)$$

We assume that the statistical behavior of the target and clutter are known, and can thus be used to construct $\boldsymbol{\mu}_0$, $\boldsymbol{\mu}_1$, $\boldsymbol{\Gamma}_0$, and

$\mathbf{\Gamma}_1$. In this case, the Neyman-Pearson detector is given by the Likelihood Ratio Test Statistic [7]:

$$\ell = (\mathbf{y} - \boldsymbol{\mu}_0)^H \mathbf{\Gamma}_0^{-1} (\mathbf{y} - \boldsymbol{\mu}_0) - (\mathbf{y} - \boldsymbol{\mu}_1)^H \mathbf{\Gamma}_1^{-1} (\mathbf{y} - \boldsymbol{\mu}_1). \tag{4}$$

A. Clutter and Target Models

We utilize the Wide-Sense Stationary Uncorrelated Scattering (WSSUS) model for multipath propagation [6]. WSSUS assumes that the multipath response comes from a linear superposition of uncorrelated echoes, and that each echo is characterized by a WSS impulse response. From this model, it follows that the clutter impulse response can be characterized as a WSS Gaussian random process with zero mean and Power Spectral Density (PSD) $P_c(\omega_q)$ [5].

Since the target is assumed to be embedded in dense scattering, we view the target impulse response as a WSS Gaussian random process with zero-mean and PSD $P_t(\omega_q)$; this is in contrast to the point target model of [5], where the target channel is modeled with a scalar complex reflectivity. Thus, the target and clutter frequency responses are drawn from the distributions:

$$H_t(\omega_q) \sim \mathcal{CN}\left(0, P_t(\omega_q)\right), \tag{5}$$

$$H_c(\omega_q) \sim \mathcal{CN}\left(0, P_c(\omega_q)\right). \tag{6}$$

The probing signal is $S(\omega_q)$. The responses of the target and clutter channels, treated as linear chanels, to the probing signal are:

$$T(\omega_q) = H_t(\omega_q)S(\omega_q), \tag{7}$$

$$C(\omega_q) = H_c(\omega_q)S(\omega_q). \tag{8}$$

The additive noise term in (1), $V(\omega_q)$, represents both noise and any incoherent interference sources; its PSD is $P_v(\omega_q)$. In the next section, we will discuss the probing signal $S(\omega_q)$, and how it is chosen under both Water Filling and TR.

III. Detection

We consider the Likelihood Ratio Test (LRT) for the model derived in Section II. This section is broken into three parts. First, we discuss the classical "water filling" approach and present the corresponding LRT. Second, we discuss the TR detection strategy and present the TR-LRT. Finally, we discuss an ideal TR strategy that provides an upper bound on the performance of TR detection.

A. Water Filling

Kay shows that, for detection of a point target in signal-dependent clutter, the optimal signal design for this problem becomes one of maximizing the metric [5]:

$$d^2 = \sum_{q=0}^{Q-1} \frac{|S(\omega_q)|^2}{P_c(\omega_q)\,|S(\omega_q)|^2 + P_v(\omega_q)}, \tag{9}$$

with the constraint that the energy in $S(\omega_q)$ is bounded by $\mathcal{E}$:

$$\sum_{q=0}^{Q-1} |S(\omega_q)|^2 \leq \mathcal{E}. \tag{10}$$

The solution is given by the classical water filling approach [5]:

$$|S(\omega)_{\text{WF}}|^2 = \max\left(\frac{\sqrt{P_v(\omega_q)/\lambda} - P_v(\omega_q)}{P_c(\omega)}, 0\right), \tag{11}$$

where λ is a parameter that satisfies the equation:

$$\sum_{q=0}^{Q-1} \max\left(\frac{\sqrt{P_v(\omega_q)/\lambda} - P_v(\omega_q)}{P_c(\omega)}, 0\right) = \mathcal{E}. \tag{12}$$

The phase of this signal can be arbitrarily chosen. This solution also arises when maximizing channel capacity for parallel channels [8]. It can be seen that this approach will concentrate energy in frequency bins where the clutter response is weaker. However, since it assumes that the target will respond uniformly across frequencies, it has the potential to significantly reduce the target response when that is not the case. For this reason, water filling is only proven to be optimal for the special case where $P_t(\omega_q) = \sigma_t^2$.

1) Conventional Detector: We consider the Likelihood Ratio Test statistic in (4). Since $S(\omega_q)$ is deterministic, we know that $Y(\omega_q)$ is distributed as a complex Gaussian with zero mean. Thus, (4) can be reduced to:

$$\ell_{\text{WF}} = \mathbf{y}^H \left(\mathbf{\Gamma}_0^{-1} - \mathbf{\Gamma}_1^{-1}\right) \mathbf{y}. \tag{13}$$

In the interest of brevity, we don't show the definition of $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_1$. We note that ℓ_{WF} is a weighted sum of chi-squared random variables, since the covariance matrices are both diagonal. Thus, while the distribution of ℓ_{WF} is not known in closed-form, it can be approximated with a Gamma distribution [9]. We will evaluate this theoretical performance and verify it with Monte Carlo simulations in Section IV.

B. Time Reversal

Time Reversal (TR) can be broken into three stages. In the first stage, a conventional transmit signal is used to probe the environment and measure the channel response. The response to this first stage is then utilized in the second stage to construct a TR waveform, which is then used in a second transmission stage to probe the scene for a target. The result of this second transmission is utilized for detection.

1) Probing Stage: To begin, we transmit a probing signal into the channel, which may or may not contain a target. For simplicity, we utilize a white probing signal:

$$\mathbf{s} = \sqrt{\mathcal{E}/Q}\,[1, \ldots, 1]^T. \tag{14}$$

Thus, the received signal, when a target is present, is:

$$\mathbf{y} = (\mathbf{H}_t + \mathbf{H}_c)\mathbf{s} + \mathbf{v}. \tag{15}$$

2) Time Reversal Waveform Redesign: This stage is straightforward. We time reverse (or phase conjugate in the frequency domain) the received signal:

$$\mathbf{s}_{\text{TR}} = k\mathbf{y}^*, \tag{16}$$

where the energy normalization factor k is defined by:

$$k^2 = \mathcal{E} / \sum_{q=0}^{Q-1} \|\mathbf{y}\|^2 . \tag{17}$$

3) Time Reversal Probing Stage: We assume that there has been a non-zero passage of time since the initial probing stage, and that the channel has partially de-correlated. We define new channel variables $\overline{H}_c(\omega_q)$, $\overline{H}_t(\omega_q)$, and $\overline{V}(\omega_q)$ drawn from the same distributions as $H_c(\omega_q)$, $H_t(\omega_q)$, and $V(\omega_q)$, respectively. We use the correlation coefficient to represent the partial coherence between the two transmission stages [7]:

$$\rho_c = \frac{E\left\{H_c(\omega_q)\overline{H}_c^*(\omega_q)\right\}}{P_c(\omega_q)}, \tag{18}$$

$$\rho_t = \frac{E\left\{H_t(\omega_q)\overline{H}_t^*(\omega_q)\right\}}{P_t(\omega_q)}. \tag{19}$$

The noise signals $V(\omega_q)$ and $\overline{V}(\omega_q)$ are, of course, independent. The received signal, following TR probing, is given by:

$$\mathbf{y}_{\text{TR}} = \left[\overline{\mathbf{H}}_t + \overline{\mathbf{H}}_c\right] \mathbf{s}_{\text{TR}} + \overline{\mathbf{v}} \tag{20}$$

4) TR Likelihood Ratio Test: We begin with the binary hypothesis in (3). We note that the TR transmit signal is partially correlated with the channel, so we must compute the conditional distribution of $\mathbf{y}_{\text{TR}}$ given $\mathbf{s}_{\text{TR}}$. The terms are jointly complex Gaussian, so we follow the form given in [7], which leads to the binary hypothesis:

$$\mathcal{H}_i : \mathbf{y}_{\text{TR}} | \mathbf{s}_{\text{TR}} \sim \mathcal{CN}\left(\boldsymbol{\mu}_{\text{TR}|i}, \boldsymbol{\Gamma}_{\text{TR}|i}\right). \tag{21}$$

In the interest of brevity, we omit the expressions for the mean and variance terms. We then construct the TR LRT from (4):

$$\begin{aligned}\ell_{\text{TR}} &= \left(\mathbf{y}_{\text{TR}} - \boldsymbol{\mu}_{\text{TR}|0}\right)^H \boldsymbol{\Gamma}_{\text{TR}|0}^{-1} \left(\mathbf{y}_{\text{TR}} - \boldsymbol{\mu}_{\text{TR}|0}\right) \\ &- \left(\mathbf{y}_{\text{TR}} - \boldsymbol{\mu}_{\text{TR}|1}\right)^H \boldsymbol{\Gamma}_{\text{TR}|1}^{-1} \left(\mathbf{y}_{\text{TR}} - \boldsymbol{\mu}_{\text{TR}|1}\right). \end{aligned} \tag{22}$$

We note that the mean terms are directly related to ρ_c and ρ_t.In other words, as $\rho_c \to 0$ and $\rho_t \to 0$, the expectation $\boldsymbol{\mu}_{\text{TR}|i} \to 0$. The test statistic ℓ_{TR} has a distribution that is not known in closed form. However, we note that when the correlation parameters equal 0 ($\rho_t = \rho_c = 0$), the distribution becomes zero-mean and the TR-LRT takes the same form as (13). Thus, in that extreme, we can use the Gamma approximation to compute theoretical performance curves.

C. Ideal Time Reversal

In an ideal world, we would learn the target response directly, instead of in the presence of noise and clutter. Thus, for comparison to an ideal scenario, we assume that the initial probing stage results in the signal $\mathbf{y} = \mathbf{H}_t\mathbf{s}$. This is equivalent to (15), with the clutter and noise contributions removed. This allows us to construct the Ideal TR signal: $\mathbf{s}_{\text{ITR}} = k\mathbf{H}_t^*\mathbf{s}^*$, with k defined in (17). Thus, the received signal is given by:

$$\mathbf{y}_{\text{ITR}} = \left[\overline{\mathbf{H}}_t + \overline{\mathbf{H}}_c\right] \mathbf{H}_t^*\mathbf{s}^* + \overline{\mathbf{v}}. \tag{23}$$

We proceed with detection in the same manner as in the standard TR case. This signal will provide an upper bound on the performance of TR since it is coherent only with the target response and not the clutter, thereby maximizing the distance between the two distributions.

IV. Simulation Results

We test the performance of the water filling and TR based detectors by simulating first a point target, then a distributed target, embedded in dense multipath. For each scenario, we use the PSDs to construct the three waveforms and conduct $MC = 10,000$ Monte Carlo trials for each waveform. The results are used to calculate the achievable P_{D} for each waveform, given a desired false alarm rate of $P_{\text{FA}} = 0.01$. In addition to the simulation results, we also compute theoretical performance with the Gamma approximation, when appropriate.

The simulations are conducted for $Q = 201$ frequency samples across a $BW = 2GHz$ band. We assume that the Target and Clutter channels lose coherence at the same rate ($\rho = \rho_c = \rho_t$). We consider the special case of white noise ($P_v(\omega_q) = \sigma_v^2$), and define the Signal-to-Noise Ratio (SNR):

$$\text{SNR} = \frac{\sum_{q=0}^{Q-1} P_t(\omega_q)}{\sigma_n^2}. \tag{24}$$

The detection results are plotted against SNR. For these plots, a shift to the left is viewed as an SNR gain. We will discuss performance comparisons in terms of these shifts.

A. Point Target

The results of the first test, a point target embedded in multipath, are given in figure 2. Point targets have an impulse response consisting of a single delta function, thus their PSD is flat. For this reason, a point target embedded in multipath will assume the same PSD as the clutter response, with an unknown scaling factor. Thus, we have:

$$P_t(\omega_q) = \sigma_t^2 P_c(\omega_q). \tag{25}$$

For this test, we set $\sigma_t^2 = 1$. The first result to note is that Time Reversal achieves a performance increase of roughly 2dB over Water Filling. This exhibits the utility of channel coherence, and the need to exploit it, if it exists. Considering the Ideal TR signal, however, we see that the presence of clutter and noise in the TR probing signal limits performance, since the Ideal TR signal shows an extra 8dB of potential gain. Moving to the independent channel scenario, however, we see a different result. The ideal and realistic TR waveforms have almost identical performance. This is because both rely on coherence with the target channel in order to provide a performance gain, and that coherence is lost in this scenario. In fact, the water filling approaches outperform TR above $P_{\text{D}} = 0.9$. This result implies that TR should not be blindly applied, and agrees with [10], which indicates that there is a value $\rho^{\dagger}$ such that TR is only beneficial if $\rho \geq \rho^{\dagger}$.

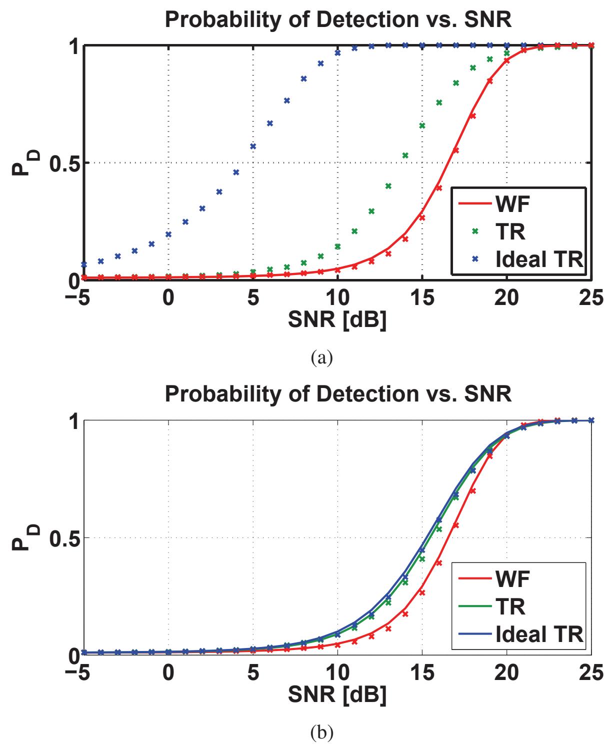

Fig. 2. Performance results (theoretical curves with markers showing simulation results) for the point target scenario. (a) Partially correlated channels ($\rho_c = \rho_t = 0.7$), (b) Independent channels ($\rho_c = \rho_t = 0$).

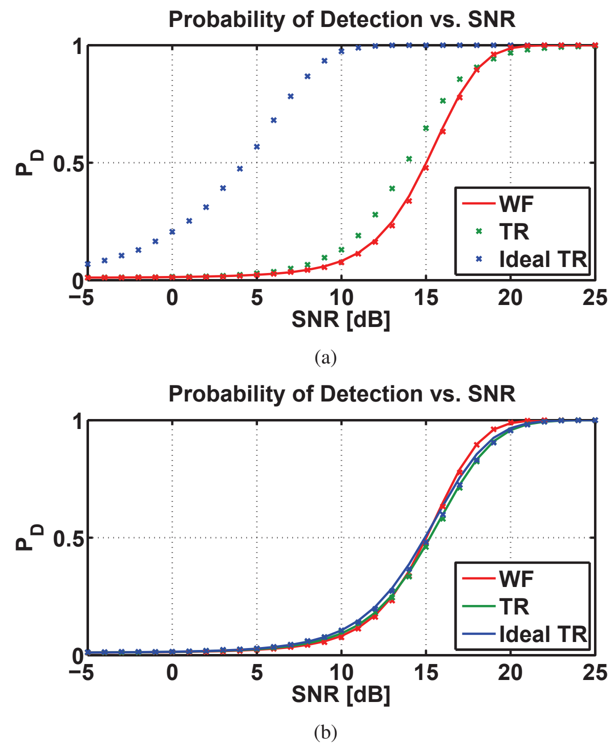

Fig. 3. Performance results (theoretical curves with markers showing simulation results) for the extended target scenario. (a) Partially correlated channels ($\rho_c = \rho_t = 0.7$), (b) Independent channels ($\rho_c = \rho_t = 0$).

B. Extended Target

To simulate an extended target (see figure 3), we took the PSDs from the point target case, and performed a random permutation of the target PSD across frequency. This permutation increased the performance of the Water Filling approach (almost 2dB of SNR gain over the point target scenario). What we see in this scenario for Time Reversal is that, while Ideal TR promises an upper bound of 10dB of gain for TR over the Water Filling approach, the realistic waveform implemented here is only 1dB above water filling. This reduction in SNR gain can be explained by the permutation of PSDs. The point target case represented a worst-case scenario for water filling, one where the strategy of minimizing the clutter response also has the effect of mitigating the target response. The distributed target, however, is a general case, where the target's PSD is not equal to the clutter PSD, and shows that the SNR gain of TR in the point target case is partially a result of improved power allocation. As with the point target, though, when the channel loses coherence, the TR waveforms no longer outperform the conventional Water Filling approach.

V. Conclusion

This paper considers the problem of detecting a target in the presence of stationary random multipath clutter. We develop a Time Reversal (TR) based approach, as an extension of prior results for deterministic clutter, and derive the LRT for this detector. We contrast this TR detector with a conventional approach that is based on the well-known "water filling." The water filling approach is the optimal strategy in the special case of a target with a flat Power Spectral Density (PSD). We show, however, that in the general case of a non-flat target PSD, TR achieves $1-2$dB of SNR gain over water filling, and that there is potential for as much as 8dB of additional gain. We show that this gain is dependent on the coherence of the channel between successive measurements.

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